

# Cell Selection Using Recursive Bipartite Matching

Anna Zakrzewska, Sarah Ruepp, Michael S. Berger

Department of Photonics Engineering

Technical University of Denmark

Ørsteds Plads 343, 2800 Kgs. Lyngby, Denmark

Email: {azak, srru, msbe}@fotonik.dtu.dk

**Abstract**—Wireless communication network consist nowadays of multiple standards, as well as cells of different sizes and coverage. Providing the best connection in such environment is a challenging task. We propose a new approach of solving the cell selection problem in heterogeneous networks. The method recursively applies efficient algorithms for bipartite graph matching to provide the final solution.

## I. INTRODUCTION

Wireless communications is the fastest developing and growing section of the telecommunications industry. Providing connectivity anytime, anywhere and from any kind of device is the trend in wireless research nowadays. Systems that will provide multimode terminals seamless operation in various Radio Access Technologies (RATs) like LTE, HSPA and WiFi, are under consideration. Moreover, the network architecture has become highly hierarchical due to introduction of smaller cells (pico, femto) served by low power base stations (BS), as presented in Fig. 1.

The problem of cell selection optimizing the usage of network resources in such a multistandard and layered heterogeneous environment becomes very complex. In this work we present an algorithm to find an optimal terminal-base station assignment maximizing the chosen utility.

## II. PROBLEM STATEMENT

In the real network deployments, a macro BS offers services to a large number of users and in a dense urban deployment macro inter-cell distance is approximately 500 m. Furthermore, there is also a number of available small cells forming additional layer of the communication system, which

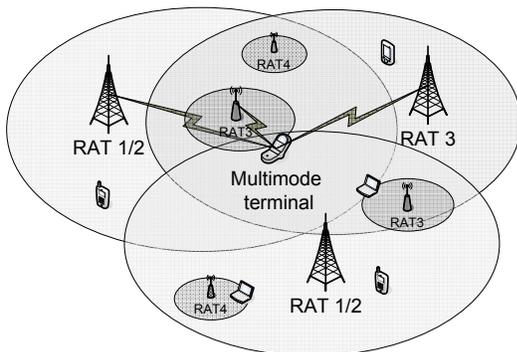


Fig. 1. Cell selection in multi-standard multi-layer network

makes the optimal cell selection more difficult. The radio environment is highly dynamic, as the transmission parameters change constantly. Here, it is assumed that:

- 1) A terminal can be connected to only one cell at a time;
- 2) Cells have a limited capacity  $c_b$  and can serve only a certain number of users;
- 3) A terminal can switch between the standards/cells during an ongoing service (seamless handover is enabled).

The goal is to provide a connection for as many terminals as possible via the most appropriate link in such a way that the capacity of a cell is not exceeded. The problem could be formulated using Integer Linear Programming as follows:

$$\max \sum_{t \in T} \sum_{j \in B} y_{tb} u_{tb} \quad (1)$$

$$\text{s.t.} \sum_{b \in B} y_{tb} = 1 \quad t \in T \quad (2)$$

$$\sum_{t \in T} y_{tb} \leq c_b \quad b \in B \quad (3)$$

where  $y_{tb}$  is a binary terminal-BS connectivity variable,  $y_{tb} \in \{0, 1\}$ . Constraints (2) allow each terminal to establish only one connection (simultaneous connections to two RATs, like HSPA and WiFi hotspot are excluded in this model), whereas constraints (3) control the number of the connections to a cell which is limited by its capacity.

With the two disjoint groups of nodes i.e. BSs and terminals, the network can be represented as a bipartite graph. Let us then consider a bipartite graph  $G = (B \cup T, U)$  with the sets defined as follows:  $B = \{1, 2, \dots, b\}$  is a set of BSs operating in one of the RATs and offering service as a macro-, pico- or femtocell.  $T = \{1, 2, \dots, t\}$  is a set of multimode terminals and  $U$  is a set of utility values  $u_{tb}$  describing suitability of a connection of terminal  $t$  to BS  $b$ . An example of a system consisting of  $|B| = 4$  BSs providing service to  $|T| = 7$  terminals is depicted in Fig. 2 where an example matching is marked in bold.

If a BS was allowed to serve only one terminal, this would be a classic general assignment problem [1], and the task would be to find a matching (collection of the edges of graph  $G$  so that no two edges share a vertex), which could be solved using Hungarian [2] or Hopcroft-Karp algorithm [3]. However, because of constraints (3), a new method needs to be applied and our proposal is presented in the next section.

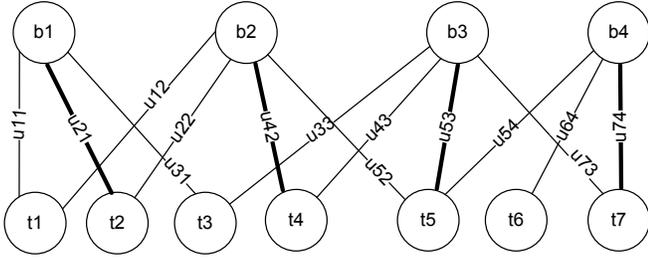


Fig. 2. Example bipartite graph with a matching

### III. RECURSIVE BIPARTITE MATCHING

Depending on the objective function, different algorithms may be used to find the optimal matching in a bipartite graph. If the number of connected terminals is to be maximized, Hopcroft-Karp algorithm would be the best choice, as it finds maximum matching in a bipartite graph at the cost  $O(m\sqrt{n})$ , where  $m$  is the number of vertices and  $n$  represents edges. On the other hand, if we consider a weighted bipartite graph as in Fig. 2, the Hungarian algorithm would provide the fastest solution at  $O(n^3)$ . A problem arises, as the two sets are of not equal cardinality  $|B| \neq |T|$  and a BS  $b$  serves more than one terminal, which contradicts with the definition of a matching.

To address this issue we propose a recursive approach to terminal-BS assignment, such that matchings found in the consecutive rounds can be combined to form the final solution of the problem. Algorithm 1 describes this mechanism.

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#### Algorithm 1 Recursive Bipartite Matching

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**Require:** Bipartite graph  $G(B \cup T, U)$

**Ensure:** Matching  $M \subseteq U$

$G' \leftarrow G$

$M \leftarrow \emptyset$

**repeat**

$G \leftarrow G'$

▷ run Hopcroft-Karp ( $G$ )

**repeat**

$P \leftarrow P_1, P_2, \dots, P_k$

$M \leftarrow M \oplus (P_1 \cup P_2 \cup \dots \cup P_k)$

**until**  $P = \emptyset$

*update*  $G'(B' \cup T', U')$

**until**  $T' = \emptyset$  or  $T = T'$

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Since it is a recursive approach, by  $G'(B' \cup T', U')$  we denote the bipartite graph that remains after removal of a matching found in  $G$ .  $B'$ ,  $T'$  and  $U'$  are the remaining subsets of unassigned resources after a partial solution is found. This means BSs that can still accept users' requests, terminals that haven't been matched and corresponding edges representing the utilities. The algorithm starts with an empty matching  $M$  which is gradually updated with the solutions of the adopted matching algorithm e.g. Hopcroft-Karp. The procedure is executed until all the terminals are assigned to a serving cell or no more matchings can be found in the remaining subset (the subset of terminals before and after calling the internal algorithm is the same). The complexity of such approach solely depends on the core algorithm used, as it will be run from  $|T| \div |B| + 1$  to maximum  $|T|$  times.

### IV. UTILITY FUNCTION

As we noted in the introduction, an optimal solution to the cell selection problem maximizes the chosen utility. Consequently, the utility definition influences the type of internal algorithm to be used in the proposed procedure. Hopcroft-Karp is suitable when the goal is to maximize the number of connected users (maximum cardinality matching), whereas Hungarian algorithm will be best in case when different connections have different weights (maximum weighted bipartite matching). Utility function can be defined in a number of ways, taking into account diverse parameters such as terminal capability, link quality (throughput, delay), cell load, cell type (macro, pico), user mobility or service cost among others [4], [5], [6], [7]. Furthermore, these factors can be taken into account with different weights summing up to 1. Definition of the utility function affects the Quality of Service (QoS), resource utilization and overall network performance resulting from a particular matching, and is therefore an interesting topic itself.

### V. CONCLUSION AND FURTHER WORK

We present a new approach to cell selection in multistandard heterogeneous networks. The idea is based on a recursive use of algorithms dedicated to find matchings in bipartite graphs. A platform for evaluation purposes has already been implemented: a dedicated network model developed in OPNET Modeler [8] has been integrated with GAMS [9] using CPLEX as IP solver [10]. We plan to compare the proposed method with ILP formulations of the problem using various network scenarios and different definitions of the utility function. Furthermore, we will study a modified version of this problem which relaxes constraints (2) and allows for simultaneous connections to multiple RATs.

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